

Mass-Inflation in Dynamical Gravitational Collapse of a Charged Scalar-Field

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Abstract

We study the inner-structure of a charged black-hole which is formed from the gravitational collapse of a self-gravitating *charged* scalar-field. Starting with a *regular* spacetime, we follow the evolution through the formation of an apparent horizon, a Cauchy horizon and a final central singularity. We find a null, *weak*, mass-inflation singularity along the Cauchy horizon, which is a precursor of a strong, *spacelike* singularity along the $r = 0$ hypersurface.

The *no-hair* theorem, introduced by Wheeler [1] in the early 1970s, states that the *external* field of a black-hole relaxes to a Kerr-Newman field characterized solely by the black-hole's mass, charge and angular-momentum. This simple picture describing the exterior of a black-hole is in dramatic contrast with its *interior*. The singularity theorems of Penrose and Hawking [2] predicts the occurrence of inevitable spacetime singularities as a result of a gravitational collapse in which a black-hole forms. According to the *weak cosmic censorship* conjecture [3] these spacetime singularities are hidden beneath the black-hole's event-horizon. However, these theorems tell us nothing about the nature of these spacetime singularities. In particular, the *final* outcome of a generic gravitational collapse is still an open question in general relativity.

Until recently, our physical intuition regarding the nature of these inner singularities was largely based on the known static or stationary black-hole solutions: Schwarzschild (*space-*

like, strong and unavoidable central singularity), Reissner-Nordström and Kerr (*timelike*, strong singularity). Further insight was gained from the work of Belinsky, Khalatnikov and Lifshitz [4] who found a strong oscillatory spacelike singularity. However, a new and drastically different picture of these inner black-hole singularities has emerged in the last few years, according to which the CH inside charged or spinning black-holes is transformed into a *null*, *weak* singularity [5–9]. The CH singularity is weak in the sense that an infalling observer which hits this null singularity experiences only a finite tidal deformation [7,8]. Nevertheless, curvature scalars (namely, the Newman-Penrose Weyl scalar Ψ_2) diverge along the CH, a phenomena known as *mass-inflation* [6]. The physical mechanism which underlies this CH singularity is actually quite simple: small perturbations, which are remnants of the gravitational collapse *outside* the collapsing object are gravitationally *blueshifted* as they propagate in the black-hole’s interior parallel to the CH [10] (the mass-inflation scenario itself includes in addition an outgoing radiation flux which irradiates the CH. This outgoing flux represents a portion of the ingoing radiation which is scattered *inside* the black-hole).

Yet, it should be emphasized that despite of the remarkable progress in our physical understanding of the *inner*-structure of black-holes, the evidence supporting the existence of a null, weak CH singularity is mostly based on *perturbative* analysis. Thus, it is of interest to perform a *full* non-linear investigation of the inner-structure of black-holes. The pioneering work of Gnedin and Gnedin [11] was a first step in this direction. They have demonstrated the existence of a central *spacelike* singularity deep inside a charged black-hole coupled to a (neutral) scalar-field. Much insight was gained from the numerical work of Brady and Smith [9] who studied the non-linear evolution of a (neutral) scalar-field on a spherical charged black-hole. These authors established the existence of a *null* mass-inflation singularity along the CH. Furthermore, they have shown that the singular CH contracts to meet the central $r = 0$ *spacelike* singularity. More recently, Burko [12] studied the same model problem. His work improve the numerical results given in [9], namely, he found a good agreement between the non-linear numerical results and the predictions of the perturbative approach.

Despite the important results achieved in these numerical investigations the mass-inflation scenario has *never* been demonstrated *explicitly* in *collapsing* situation. It should be emphasized that these numerical works begin on a *singular* Reissner-Nordström spacetime and the black-hole formation was *not* calculated there. The main goal of this paper is to demonstrate *explicitly* that mass-inflation takes place during a dynamical charged gravitational collapse.

We consider the gravitational collapse of a self-gravitating *charged* scalar-field ϕ . While charged collapse is not expected in a realistic gravitational collapse, it is generally accepted that the similarity between the inner structure of a Reissner-Nordström black-hole and a Kerr black-hole indicates that a charged collapse might be a simple (spherical) toy model for the more realistic generic rotating collapse. In previous papers we have investigated the *linear* [13,14] and *non-linear* [15] evolution of a *charged* scalar-field *outside* a charged black-hole. The results given in these papers, and in particular the existence of oscillatory inverse *power-law* charged tails along the black-hole outer horizon suggest the occurrence of mass-inflation along the CH of a *dynamically* formed charged black-hole. Thus, this model is suitable to establish our main goal.

Our scheme is based on *double null* coordinates. This allows us to begin with *regular* initial spacetime (at approximately past null infinity), calculate the *formation* of the black-hole's event horizon, and follow the evolution *inside* the black-hole all the way to the central singularity, which is formed *during* the collapse. Thus, this numerical scheme makes it possible to test *directly* (and for the first time) the conjecture that the mass-inflation scenario is an inevitable feature of a generic gravitational collapse.

The physical model is described by the coupled Einstein-Maxwell-charged scalar equations. We express the metric of a spherically symmetric spacetime in the form [16]

$$ds^2 = -\alpha(u, v)^2 dudv + r(u, v)^2 d\Omega^2 , \quad (1)$$

where u is a retarded time null coordinate and v is an advanced time null coordinate. We fix the axis $r = 0$ to be along $u = v$. The remaining coordinate freedom is the freedom of

the choice of v along some future null cone, i.e. along some fixed $u = \text{const}$ outgoing null ray. It should be noted that for $v \gg M$ our null ingoing coordinate v is proportional to the Eddington-Finkelstein null ingoing coordinate v_e . Following Hamade and Stewart [17] we formulate the problem as a system of *first*-order coupled PDEs. To do so we define auxiliary variables d, f, g, s, x and y :

$$d = \frac{\alpha_v}{\alpha}, \quad f = r_u, \quad g = r_v, \quad s = \sqrt{4\pi}\phi, \quad x = s_u, \quad y = s_v. \quad (2)$$

Using these variables one can generalize the neutral Hamade and Stewart scheme [17]. In terms of these new variables the Einstein equations expand to

$$E1 \equiv d_u - \frac{fg}{r^2} - \frac{\alpha^2}{4r^2} + \frac{\alpha^2 q^2}{2r^4} + \frac{1}{2}(xy^* + x^*y) + \frac{1}{2}iea(sy^* - s^*y) = 0, \quad (3)$$

$$E2 \equiv rf_v + fg + \frac{1}{4}\alpha^2 - \frac{\alpha^2 q^2}{4r^2} = 0 \quad (4)$$

and

$$E3 \equiv g_v - 2dg + ry^*y = 0. \quad (5)$$

The electromagnetic potential $a(u, v) \equiv A_0$ is given by the Maxwell equations

$$M1 \equiv a_v - \frac{\alpha^2 q}{2r^2} = 0, \quad (6)$$

where the charge $q(u, v)$ is given by

$$M2 \equiv q_v - ier^2(s^*y - sy^*) = 0. \quad (7)$$

The Hawking mass function $m(u, v)$ is given by

$$m = \frac{r}{2} \left(1 + \frac{q^2}{r^2} + \frac{4}{\alpha^2} r_u r_v \right). \quad (8)$$

Finally, the wave-equation for the charged scalar-field becomes

$$S1 \equiv ry_u + fy + gx + ieary + ieags + \frac{ie}{4r} \alpha^2 qs = 0. \quad (9)$$

The definition-equation (2) yields

$$D1 \equiv d - \frac{\alpha_v}{\alpha} = 0, \quad D2 \equiv g - r_v = 0, \quad D3 \equiv y - s_v = 0. \quad (10)$$

The initial conditions include the specification of $y(0, v)$ and $d(0, v)$ along an outgoing $u = 0$ null ray. We assume $d(0, v) = 0$, which fixes the remaining freedom in the coordinates. The boundary conditions on the axis $r = 0$ ($u = v$) are $g = -f = \frac{1}{2}\alpha$, $x = y$, $a = q = 0$ and $\alpha_r = s_r = 0$ (on axis).

The evolution of the quantities d and y are determined by Eqs. E1 and S1, respectively. We then integrate Eq. D1 outwards from the axis along an outgoing ($u = \text{const}$) null ray to find α . Eqs. E3 and D2 are used to obtain g and r . Finally, we evaluate the quantities s, q, a, f and x by integrating Eqs. D3, M2, M1, E2 and S1, respectively (x is evaluated from S1 using the relation $x_v = y_u$). The integration in the u -direction is carried out using a fifth-order Runge-Kutta method, while the integrals in the v -direction are discretized using the three-point Simpson method [18].

Figure 1 displays the radius $r(u, v)$ as a function of the ingoing null coordinate v along a sequence of outgoing ($u = \text{const}$) null rays. All the outgoing null rays originate from the *non-singular* axis $r = 0$, i.e. we start with a *regular* spacetime (this situation is in contrast with previous numerical works, where the evolution begins on a Reissner-Nordström spacetime). One can distinguish between *three* types of outgoing null rays in the rv plane: (i) The outer-most (small- u) rays, which *escape* to infinity. (ii) The intermediate outgoing null rays approach a fixed radius $r_{CH}(u)$ at late-times $v \rightarrow \infty$. This indicates the existence of a CH in these spacetimes. (iii) The inner-most (large- u) rays, which originate on the *non-singular* axis $r = 0$ and terminate at the *singular* section of the $r = 0$ hypersurface. These outgoing rays reach the $r = 0$ singularity in a *finite* v , *without* intersecting the CH. This situation is in contrast with the Reissner-Nordström spacetime, in which it is well-known that all the outgoing null rays which originate inside the black-hole intersect the CH. One should also note that in contrast with the Reissner-Nordström spacetime, where the CH is a *stationary* null hypersurface, here $r_{CH}(u)$ depends on the outgoing null coordinate u , i.e. the

CH *contracts* [9]. This dramatic difference in the causal structure of the present *collapsing* spacetime compared with the Reissner-Nordström spacetime is attributed to the outgoing flux of energy-momentum carried by the charged scalar-field.

To understand better the causal structure of our dynamical spacetime we display in Fig. 2. the contour lines of $r(u, v)$ in the vu -plane. The outer most contour line corresponds to $r = 0$, where its left section (a straight line $u = v$) is the *non-singular* axis, and its right section corresponds to the central singularity at $r = 0$. It should be emphasized that this central singularity forms *during* the gravitational collapse. The singularity at the $r = 0$ hypersurface is clearly a *spacelike* one for r_v is *negative* along this section. The vanishing of r_v indicates the existence of an apparent horizon (which is first formed at $u \approx 1$ for this specific solution). The CH itself is a *null* hypersurface which is located at $v \rightarrow \infty$. Its existence is indicated by the fact that the intermediate outgoing null rays (in the range $1 \lesssim u \lesssim 2.1$ for this specific solution) terminate at a finite (u -dependent) radius $r_{CH}(u)$. The singular CH contracts to meet the central ($r = 0$) spacelike singularity (along the $u \simeq 2.1$ outgoing null ray). Thus, the *null* CH singularity is a precursor of the final *spacelike* singularity along the $r = 0$ hypersurface [9].

The behaviour of the mass function $m(u, v)$ along the outgoing null rays is displayed in the top panel of Fig. 3. This figure establish *explicitly* the exponential divergence of the mass function (and curvature) along the CH in a *dynamically* collapsing spacetime. To our knowledge, this is the *first* explicit demonstration of the mass-inflation scenario in a *collapsing* situation starting from a *regular* spacetime. The mass function increases not only along the outgoing ($u=\text{const}$) null rays (as v increases) but also along ingoing ($v=\text{const}$) null rays (as u increases).

The *weakness* of the null mass-inflation singularity was first predicted by Ori [7,8]. It is demonstrated in the bottom panel of Fig. 3. This figure displays the metric function g_{uv} along an outgoing null ray, where V is a Kruskal-like ingoing null coordinate. Clearly, g_{uv} approaches a *finite* value as the CH is approached ($V \rightarrow 0$). This confirms the analytical analysis of Ori, according to which a suitable coordinate transformation can produce a

non-singular metric.

Figure 4 displays the Ricci curvature scalar (on the axis):

$$R(u, v) = -\frac{8}{\alpha^2} [Re(y^*x) + eaIm(s^*y)] , \quad (11)$$

as a function of T , the proper time on axis [17]:

$$T(u) = \int_0^u \alpha(w, w) dw . \quad (12)$$

The curvature on the axis diverges in a *finite* proper time. Thus, the *initially regular* axis is replaced by a (spacelike) *singularity* along the $r = 0$ hypersurface.

In summary, we have studied the gravitational collapse of a self-gravitating *charged* scalar-field. We calculate the formation of an apparent horizon, followed by a *weak*, null, *mass-inflation* singularity along the *contracting* CH, which precedes a *strong*, *spacelike* singularity along the $r = 0$ hypersurface. Our results give a first *explicit* confirmation of the mass-inflation scenario in a dynamical collapse that begins with *regular* initial conditions.

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FIGURES

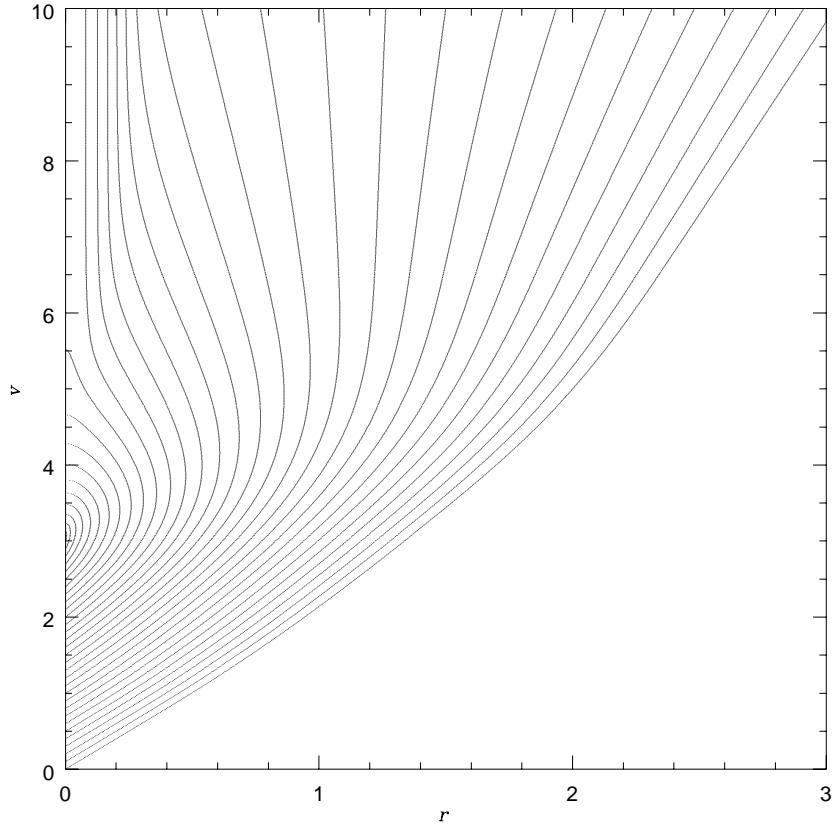


FIG. 1. Null rays in the rv -plane. One can distinguish between three types of outgoing null rays: The outer-most, which escape to infinity, the inner-most which terminate at the singular section of the $r = 0$ hypersurface and the intermediate outgoing null rays which approach a (u -dependent) finite radius, indicating the existence of a CH. All the null rays originate from the *non-singular* axis $r = 0$.

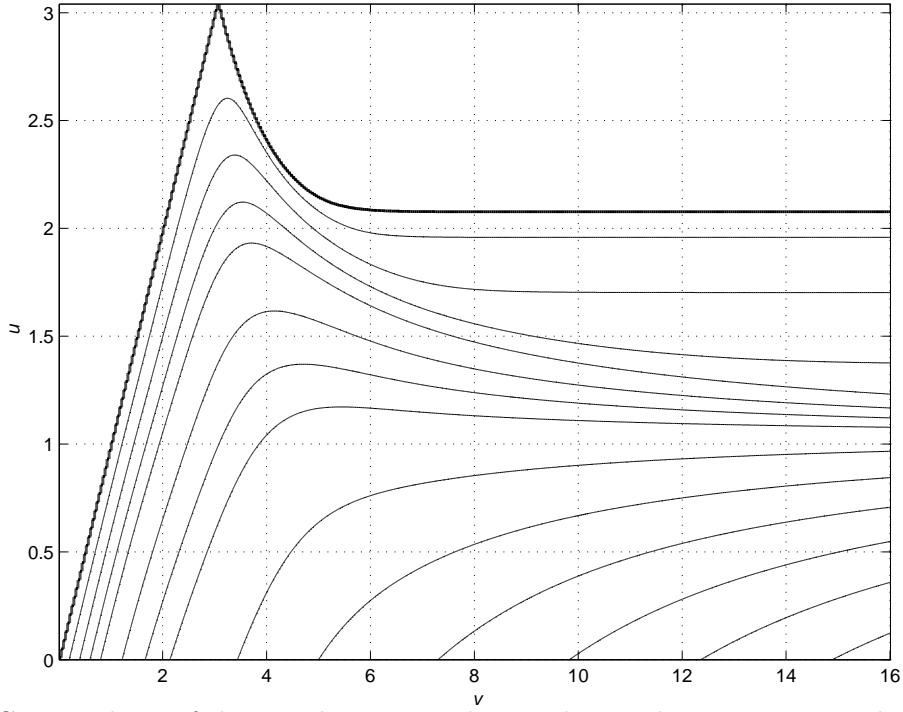


FIG. 2. Contour lines of the coordinate r in the vu -plane. The $r = 0$ contour line is indicated by a thicker curve. Its left section ($u = v$) represents the *non-singular axis*, while its right section corresponds to the central *spacelike singularity*. The apparent horizon is indicated by the vanishing of r_v . The (singular) CH (a null hypersurface, located at $v \rightarrow \infty$, and indicated by the approach of outgoing null-rays to *finite* values of r) *contracts* to meet the *central* spacelike singularity (in a *finite* proper time).

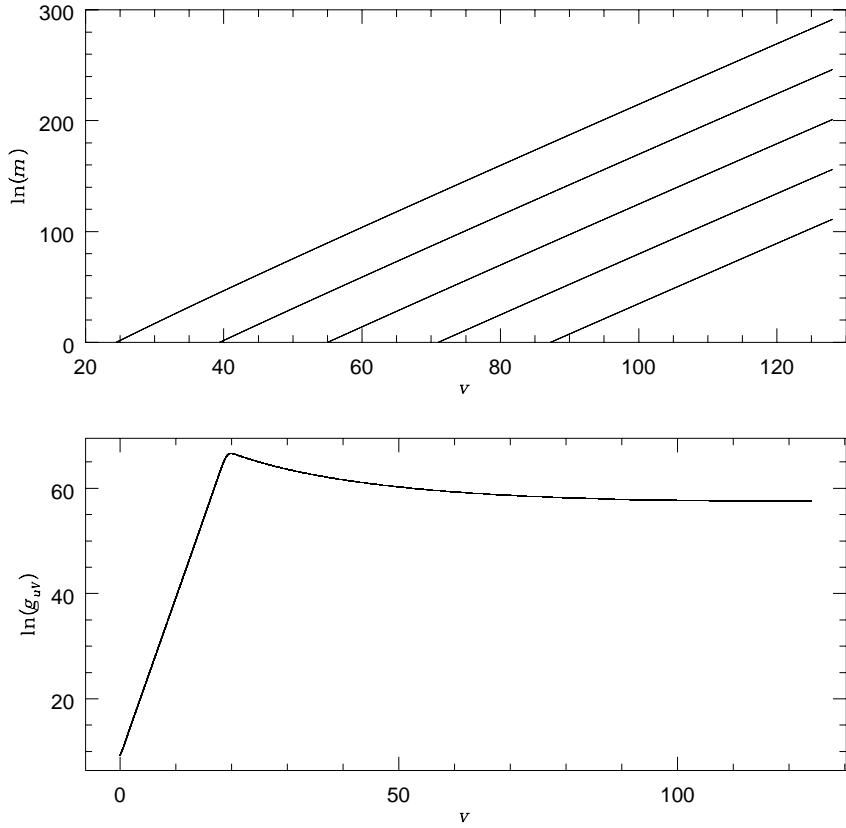


FIG. 3. The CH singularity. The top panel displays $\ln(m)$ vs. advanced time v , along a sequence of outgoing null rays. The exponential growth of the mass-function demonstrates the appearance of the mass-inflation scenario [6]. The bottom panel displays the metric function g_{uv} along an outgoing null ray. The finite value approached by the metric functions is in agreement with the simplified model of Ori [7,8], and demonstrate the *weakness* of the null mass-inflation singularity.

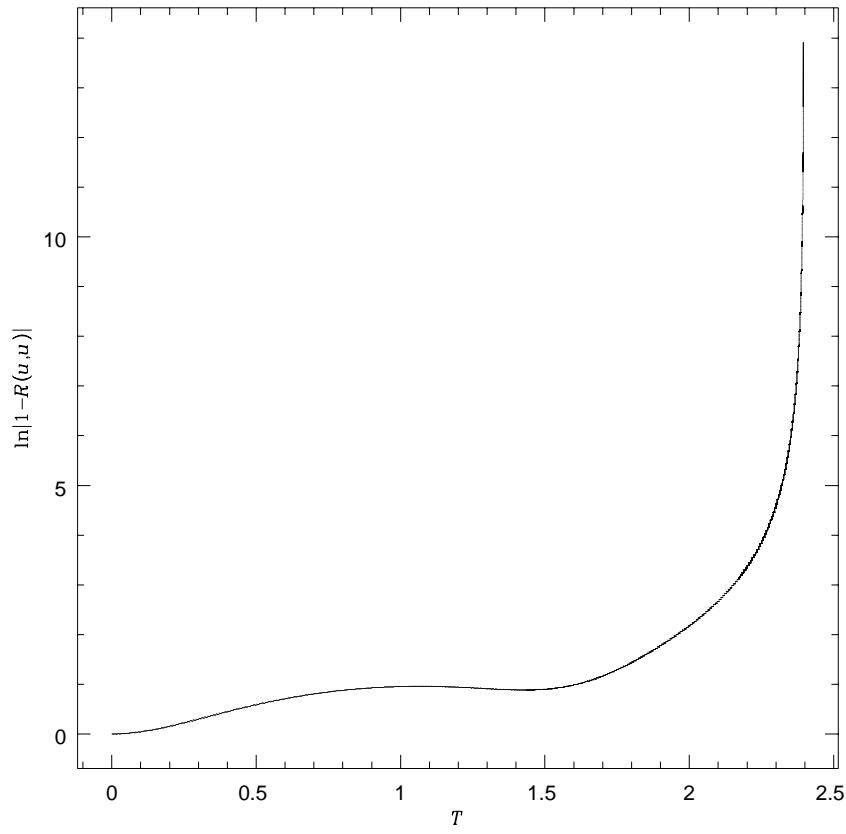


FIG. 4. The Ricci curvature scalar $R(u, u)$ as a function of the proper time on axis. The divergence of the curvature on the axis indicates that the *initially regular* axis is replaced in a *finite* proper time by a central (spacelike) *singularity*.